

Maths Revision Test – Functions, Calculus

The following maths revision test is aimed at students who have not studied maths for some time. The test will help you identify any areas that need revising. You can return your answers to Dan Altmann or Tim Collier during the orientation week. There will be some maths revision sessions on Tuesday of the orientation week. These sessions are again aimed at students who have not studied maths for some time. If you would like further maths revision material ahead of the course starting please contact Dan Altmann. Please also see the end of this document for suggested maths revision help.

1 Functions

1.1 Sketch the following functions and identify their domain and range:

- (a) $f(x) = 2$
- (b) $f(x) = \sqrt{x}$
- (c) $f(x) = 1/x$
- (d) $f(x) = \log_e(x)$
- (e) $f(x) = \exp(x)$
- (f) $f(x) = |x|$
- (g) $f(x) = x^2$

PS: domain = all possible values of x , so for (a) domain = $(-\infty, \infty)$
 range = all possible values of $f(x)$, so for (a) range = 2

1.2 Find the roots (ie solutions) of the following quadratic equation: $x^2 + x - 6 = 0$

1.3 Multiple choice on the exponential rules:
 (PS: The exponential rules are enclosed on a separate sheet)

a^0	=	0	1	a	None of these
3^2	=	6	8	9	None of these
1^3	=	1	3	$1/3$	None of these
2^{-3}	=	-6	$1/8$	-9	None of these
$4^3/4^5$	=	4^8	4^{-8}	16^{-1}	None of these
$(3^{-3})^3$	=	1	3^{-9}	3^{-27}	None of these
$5^2/3^2$	=	$(5/3)^2$	$(5/3)^{-1}$	5^{-6}	None of these
4^3	=	12	16	2^6	None of these
$27^{-2/3}$	=	$1/18$	$1/81$	$1/9$	None of these

1.4 Multiple choice on the rules for logarithms:

(PS: The rules for logarithms are enclosed on a separate sheet)

$\log_{10}(10^n) =$	$10n$	n	10^n	$10/n$
$\log_{10}(10^4/10^{-3}) =$	10^7	1	10	7
$1/2\log_{10}(16) =$	4	8	$\log_{10}(2)$	$\log_{10}(4)$
$\log_{10}[\log_{10}(10)] =$	10	1	0	-1
$\log_{10}(1000)/\log_{10}(100) =$	$3/2$	1	-1	10

1.5 Using the rules for the summation and product functions (enclosed), show that

$$\log \left\{ \prod_{i=1}^n p^{x_i} (1-p)^{y_i} \right\} = (\sum_i x_i) \log(p) + (\sum_i y_i) \log(1-p)$$

(\sum_i is just a quick way of saying: sum over all the i (ie in this case 1 to n)).

2 Differential Calculus

2.1 Differentiate the following functions:

(PS: The differentiation rules are enclosed on a separate sheet)

(a) $f(x) = x^3$

(b) $f(x) = x^{-7}$

(c) $f(x) = \sqrt{1+x^2}$

(d) $f(x) = \frac{1}{\log_e(x)}$

(e) $f(x) = \exp(4x)$

(f) $f(x) = \frac{1+x}{x^2}$

2.2 Find the turning points of the function $f(x) = 8x + (2/x)$ (that is the values of x at which $f'(x) = 0$). Which is a minimum and which is a maximum?

2.3 if $L = \frac{-(x-\mu)^2}{s} - \log(s)$ find

(a) $\frac{\partial L}{\partial s}$ (b) $\frac{\partial L}{\partial \mu}$

(c) $\frac{\partial^2 L}{\partial s^2}$ (d) $\frac{\partial^2 L}{\partial s \partial \mu}$

(e) $\frac{\partial^2 L}{\partial \mu^2}$ (f) $\frac{\partial^2 L}{\partial \mu \partial s}$

Notice that: $\frac{\partial L}{\partial s}$ is the partial differentiation of the function L with respect to s . This means that we should differentiate L with respect to s and consider all the other variables (ie x and μ) in the function L as constant. $\frac{\partial^2 L}{\partial s^2}$ is the second partial differentiation of L with respect to s . $\frac{\partial^2 L}{\partial s^2} = \frac{\partial}{\partial s} \left(\frac{\partial L}{\partial s} \right)$, which means that we want to partially differentiate the function $\left(\frac{\partial L}{\partial s} \right)$ with respect to s . Similarly $\frac{\partial^2 L}{\partial s \partial \mu} = \frac{\partial}{\partial s} \left(\frac{\partial L}{\partial \mu} \right)$, which means that we want to partially differentiate the function $\left(\frac{\partial L}{\partial \mu} \right)$ with respect to s .

3 Integration

(PS: The integration rules are enclosed on a separate sheet)

3.1 $\int 3x^4 dx = ?$

3.2 $\int \frac{1}{x} dx = ?$

3.3 $\int \frac{x}{x^2 + 4} dx = ?$

3.4 Find the area under the curve $y = x^2$ (i) between $x=0$ and $x=3$, (ii) between $x=0$ and $x=-3$.

Exponential Rules

$$a^m \times a^n = a^{m+n}$$

$$a^{-m} = 1/a^m$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^m = a^m b^m$$

$$a^0 = 1$$

Rules for Logarithms

$$\log(ab) = \log(a) + \log(b)$$

$$\log(a/b) = \log(a) - \log(b)$$

$$\log(a^n) = n \log(a)$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

Rules for the summation and product functions

$$\prod_{i=1}^n x_i = (x_1 \cdot x_2 \cdot x_3 \cdot \dots \cdot x_n) \quad (\text{ie the product of } x_1 \text{ } x_2 \text{ until } x_n)$$

$$\prod_{i=1}^n ax_i = a^n \prod_{i=1}^n x_i$$

$$\sum_{i=1}^n x_i = (x_1 + x_2 + x_3 + \dots + x_n) \quad (\text{ie the sum of } x_1 \text{ } x_2 \text{ until } x_n)$$

$$\sum_{i=1}^n ax_i = a \sum_{i=1}^n x_i$$

$$\sum_{i=1}^n a = na$$

Some Rules for Differentiation

$$\frac{d}{dx} a = 0$$

$$\frac{d}{dx} ax = a$$

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{d}{dx} \log_e(x) = \frac{1}{x}$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} e^{F(x)} = \frac{dF(x)}{dx} e^{F(x)} = F'(x) e^{F(x)}$$

$$\frac{d}{dx} (F(x) + L(x)) = F'(x) + L'(x)$$

$$\frac{d}{dx} (F(x) \cdot L(x)) = L(x) \cdot F'(x) + F(x) \cdot L'(x)$$

$$\frac{d}{dx} (F(x)/L(x)) = \frac{F'(x)}{L(x)} - \frac{F(x) \cdot L'(x)}{[L(x)]^2} \quad \text{or} = \frac{F'(x)L(x) - L'(x)F(x)}{[L(x)]^2}$$

‘Chain rule’: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Some Important Integrals

$$\int a \, dx = ax$$

$$\int a f(x) \, dx = a \int f(x) \, dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

$$\int F'(x) [F(x)]^n \, dx = \frac{[F(x)]^{n+1}}{n+1}$$

$$\int e^x \, dx = e^x$$

$$\int F'(x) e^{F(x)} \, dx = e^{F(x)}$$

$$\int \frac{1}{x} \, dx = \log_e(x)$$

$$\int \frac{F'(x)}{F(x)} \, dx = \log_e(F(x))$$

$$\int (F(x) + L(x)) \, dx = \int F(x) \, dx + \int L(x) \, dx$$

Integrating by parts: $\int_a^b u \frac{dv}{dx} \, dx = [uv]_a^b - \int_a^b v \frac{du}{dx} \, dx$

Further material: potentially useful websites:

1) <http://www.bbc.co.uk/education/asguru/maths/>

Quite nice explanations of differentiation and integration. Instant setting up of login and password to review your 'progress'.

2) <http://www.univie.ac.at/future.media/moe/tests.html>

Has some exercises on differentiation and other topics which may be useful.

3) <http://www.tech.plym.ac.uk/maths/resources/PDFLaTeX/logs.pdf>

Useful material on logarithms. It's part of the following website which contains other helpful stuff:

<http://www.tech.plym.ac.uk/maths/resources/PDFLaTeX/mathaid.html>

Books:

Quick Calculus 2nd Edition: A self-teaching guide by Kleppner & Ramsey; published by Wiley.

Quite a few students have used this book successfully in the past. It contains introductory chapters on basic algebra etc., so it covers most relevant material (but it has no material on matrices).

The following book is a bit more expensive (approx £30), but very good and covers everything you'll need:

Engineering Mathematics, K.A.Stroud and Dexter J. Booth. Published by Palgrave
6th Edition. Publication Date: 25 Jan 2007 | ISBN-10: 1403942463 | ISBN-13: 978-1403942463