Maths Revision – Functions, Calculus

1 Functions

1.1 Sketch the following functions and identify their domain and range:

(a) $f(x) = 2$
(b) $f(\mathrm{x}) = \sqrt{x}$
(c) $f(x) = 1/x$
(d) $f(x) = \log_e(x)$
(e) $f(x) = \exp(x)$
(f) $f(x) = |x|$
(g) $f(x) = x^2$

PS: domain = all possible values of x, so for (a) domain = $(-\infty,\infty)$
range = all possible values of $f(x)$, so for (a) range = 2

1.2 Find the roots (ie solutions) of the following quadratic equation: $x^2 + x - 6 = 0$

1.3 Multiple choice on the exponential rules:
(PS: The exponential rules are enclosed on a separate sheet)

a^0	=	0	1	a	None of these
3^2	=	6	8	9	None of these
1^3	=	1	3	1/3	None of these
2^{-3}	=	-6	1/8	-9	None of these
$4^3/4^5$	=	4^8	4^{-8}	16^{-1}	None of these
$(3^{-3})^3$	=	1	3^{-9}	3^{-27}	None of these
$5^2/3^2$	=	$(5/3)^2$	$(5/3)^{-1}$	5^{-6}	None of these
4^3	=	12	16	2^6	None of these
$27^{-2/3}$	=	1/18	1/81	1/9	None of these

1.4 Multiple choice on the rules for logarithms:
(PS: The rules for logarithms are enclosed on a separate sheet)

$\log_{10}(10^n)$ =	$10n$	n	10^n	$10/n$
$\log_{10}(10^4/10^{-3})$ =	10^7	1	10	7
$1/2\log_{10}(16)$ =	4	8	$\log_{10}(2)$	$\log_{10}(4)$
$\log_{10}[\log_{10}(10)]$ =	10	1	0	-1
$\log_{10}(1000)/\log_{10}(100)$ =	3/2	1	-1	10

1.5 Using the rules for the summation and product functions (enclosed), show that

$$\log\left\{\prod_{i=1}^{n} p^{x_i}(1-p)^{y_i}\right\} = (\Sigma_i x_i)\log(p) + (\Sigma_i y_i)\log(1-p)$$

(Σ_i is just a quick way of saying: sum over all the i (ie in this case 1 to n).

Differential Calculus

2.1 Differentiate the following functions:
(PS: The differentiation rules are enclosed on a separate sheet)

(a) $f(x) = x^3$
(b) $f(x) = x^{-7}$
(c) $f(x) = \sqrt{1+x^2}$
(d) $f(x) = \dfrac{1}{\log_e(x)}$
(e) $f(x) = \exp(4x)$
(f) $f(x) = \dfrac{1+x}{x^2}$

2.2 Find the turning points of the function $f(x) = 8x + (2/x)$ (that is the values of x at which $f'(x) = 0$). Which is a minimum and which is a maximum?

2.3 if $L = \frac{-(x-\mu)^2}{s} - \log(s)$ find

(a) $\frac{\partial L}{\partial s}$ (b) $\frac{\partial L}{\partial \mu}$

(c) $\frac{\partial^2 L}{\partial s^2}$ (d) $\frac{\partial^2 L}{\partial s \partial \mu}$

(e) $\frac{\partial^2 L}{\partial \mu^2}$ (f) $\frac{\partial^2 L}{\partial \mu \partial s}$

Notice that: $\frac{\partial L}{\partial s}$ is the partial differentiation of the function *L* with respect to s. This means that we should differentiate *L* with respect to s and consider all the other variables (ie *x* and μ) in the function *L* as constant. $\frac{\partial^2 L}{\partial s^2}$ is the second partial differentiation of *L* with respect to s. $\frac{\partial^2 L}{\partial s^2} = \frac{\partial}{\partial s}\left(\frac{\partial L}{\partial s}\right)$, which means that we want to partially differentiate the function ($\frac{\partial L}{\partial s}$) with respect to *s*. Similarly $\frac{\partial^2 L}{\partial s \partial \mu} = \frac{\partial}{\partial s}\left(\frac{\partial L}{\partial \mu}\right)$, which means that we want to partially differentiate the function ($\frac{\partial L}{\partial \mu}$) with respect to *s*.

3 Integration

(PS: The integration rules are enclosed on a separate sheet)

3.1 $\int 3x^4 dx = \ ?$

3.2 $\int \frac{1}{x} dx = ?$

3.3 $\int \frac{x}{x^2+4} dx = ?$

3.4 Find the area under the curve y = x^2 (i) between *x*=0 and *x*=3, (ii) between *x*=0 and *x*=-3.

Exponential Rules

$a^m \; x \; a^n = a^{m+n}$

$a^{-m} = 1/a^m$

$\frac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{mn}$

$(ab)^m = a^m b^m$

$a^0 = 1$

Rules for Logarithms

$\log(ab) = \log(a) + \log(b)$

$\log(a/b) = \log(a) - \log(b)$

$\log(a^n) = n\log(a)$

$\log_a a = 1$

$\log_a 1 = 0$

Rules for the summation and product functions

$\prod_{i=1}^{n} x_i = (x_1 \,.\, x_2 \,.\, x_3 \,\ldots.. x_n)$ (ie the product of x_1 x_2 until x_n)

$\prod_{i=1}^{n} ax_i = a^n \prod_{i=1}^{n} x_i$

$\sum_{i=1}^{n} x_i = (x_1 + x_2 + x_3 + \ldots. + x_n)$ (ie the sum of x_1 x_2 until x_n)

$\sum_{i=1}^{n} ax_i = a\sum_{i=1}^{n} x_i$

$\sum_{i=1}^{n} a = na$

Some Rules for Differentiation

$\frac{d}{dx} a = 0$

$\frac{d}{dx} ax = a$

$\frac{d}{dx} x^n = n\, x^{n-1}$

$\frac{d}{dx} \log_e(x) = \frac{1}{x}$

$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx} e^{F(x)} = \frac{dF(x)}{dx} e^{F(x)} = F'(x)\, e^{F(x)}$

$\frac{d}{dx}(F(x)+L(x)) = F'(x) + L'(x)$

$\frac{d}{dx}(F(x) \,.\, L(x)) = L(x).F'(x) + F(x).L'(x)$

$\frac{d}{dx}(F(x)/L(x)) = \frac{F'(x)}{L(x)} - \frac{F(x).L'(x)}{[L(x)]^2}$ or $= \frac{F'(x)L(x) - L'(x)F(x)}{[L(x)]^2}$

'Chain rule': $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

<u>Some Important Integrals</u>

$\int a\, dx = ax$

$\int a\, f(x)\, dx = a \int f(x)\, dx$

$\int x^n\, dx = \frac{x^{n+1}}{n+1}$

$\int F'(x)\, [F(x)]^n\, dx = \frac{[F(x)]^{n+1}}{n+1}$

$\int e^x\, dx = e^x$

$\int F'(x)\, e^{F(x)}\, dx = e^{F(x)}$

$\int \frac{1}{x}\, dx = \log_e(x)$

$\int \frac{F'(x)}{F(x)}\, dx = \log_e(F(x))$

$\int (F(x) + L(x))\, dx = \int F(x)\, dx + \int L(x)\, dx$

Integrating by parts: $\int_a^b u \frac{dv}{dx} dx = [uv]_a^b - \int_a^b v \frac{du}{dx} dx$

Further material: potentially useful websites:

1) http://www.mathsrevision.net/alevel/

Quite nice explanations of differentiation and integration.

2) http://www.univie.ac.at/future.media/moe/tests.html

Has some exercises on differentiation and other topics which may be useful.

3) http://www.tech.plym.ac.uk/maths/resources/PDFLaTex/logs.pdf

Useful material on logarithms. It's part of the following website which contains other helpful stuff:

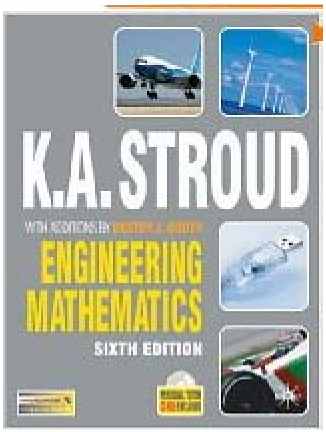
http://www.tech.plym.ac.uk/maths/resources/PDFLaTex/mathaid.html

Books:

Quick Calculus 2nd Edition: A self-teaching guide by Kleppner & Ramsey; published by Wiley.

Quite a few students have used the above book successfully in the past. It contains introductory chapters on basic algebra etc., so it covers most relevant material (but it has no material on matrices).

The following book is a bit more expensive (approx £35), but very good and covers everything you'll need:



Engineering Mathiematics
By K.A.Stroud and Dexter J. Booth. Published by Palgrave
6th Edition.