Technical Note:  
Using life table methods to calculate QALY losses from deaths: with application to COVID-19  
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First the standard approach to estimating life-expectancy is outlined with a focus on conditional life-expectancy having reached a given age. We then demonstrate how this standard approach is easily adapted to adjust for both morbidity and mortality effects of comorbidity, to give quality adjusted life-expectancy, before applying discounting to give the potential discounted QALY loss associated with a death at any given age.

2.1 Standard life table approach to estimating life-expectancy

Life tables are produced nationally and show the numbers of people dying in one-year age bands across a population. We start by defining \( q(x) \) as the probability of dying between ages \( x \) and \( x + 1 \). From this we can calculate \( l(x) \), for a reference population of 100,000, the number surviving to age \( x \geq 1 \) as:

\[
l(x) = 100,000 \times \prod_{a=1}^{x} \{1 - q(a)\}
\]

where \( l(0) = 100,000 \) by definition.

We now define \( L(x) \) as the person-years lived between ages \( x \) and \( x + 1 \) for \( x \geq 1 \):

\[
L(x) = \frac{l(x) + l(x + 1)}{2},
\]

(assuming a uniform distribution of death during the year) and the total number of person-years lived above age \( x \) as:

\[
T(x) = \sum_{u=x}^{\omega} L(u)
\]

where \( \omega \) is the upper bound of life-expectancy reported in the life table.

Now we calculate the life expectancy at age \( x \) as

\[
LE(x) = \frac{T(x)}{l(x)}
\]

2.2 Adjusting for comorbidity, quality of life and time preference

Three steps to adjusting the standard method are outlined below in order to introduce: 1) the mortality impacts of comorbidity on life-expectancy; 2) quality of life adjustment to estimate QALYs; and 3) discounting.
Comorbidities can increase a subject’s risk of death. In epidemiology, the standardized mortality ratio (SMR) summarizes how a given comorbidity can increase the risk of dying. However, applying SMR directly to the probability of death within a period would risk the probability exceeding one, especially for older ages. We therefore estimate the underlying instantaneous death rate, 
\[ d(x) = -\ln(1 - q(x)), \]
that corresponds to the per period death probability, \( q(x) \), and apply an SMR parameter to this underlying rate. This gives the equation for the reference population surviving to age \( x \), \( 1 \leq x < \omega \) to give:
\[
l_s(x) = 100,000 \times \prod_{\alpha=1}^{x} e^{-d(\alpha) \cdot SMR} \]
with \( L_s(x) \) the average of the adjacent as previously defined.

Next, we adjust for health-related quality of life by age. Standard population norm tables have been published for EQ-5D tariff values that can be used to adjust life-years to give QALYs for many different jurisdictions (Janssen B & Szende A, 2014). These tables give the population average quality of life tariff as a function of age \( x \), \( Q(x) \). Multiplying \( L(x) \) by \( Q(x) \) and an additional parameter to account for comorbidity impacts on quality of life, \( qCM \), allows the calculation of quality-adjusted \( T(x) \) and dividing by \( l_s(x) \) gives the quality-adjusted life-expectancy (QALE) at age \( x \):
\[ QALE(x) = \sum_{u=x}^{\omega} L_s(u) \cdot Q(u) \cdot qCM \cdot l_s(x) \]

The final step in providing an estimate of QALYs lost associated with a premature death at age \( x \) is to apply a discount rate \( r \) to account for the relative value of life years experienced in the future relative to the present:
\[ dQALY(x) = \sum_{u=x}^{\omega} L(u) \cdot Q(u) \cdot qCM \cdot (1 + r)^{-(u-x)} \cdot l_s(x) \]